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THE NON-EQUILIBRIUM RELAXATION MODEL FOR ONE-DIMENSIONAL FLASHING LIQUID FLOW

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Abstract—This paper presents a correlation for the relaxation time which is a closure law for the homogeneous relaxation model (HRM). The HRM takes into account the non-equilibrium evaporation leading to the metastable liquid conditions. The purpose of this paper is to present the closed HRM and to show comparisons between its results and some available data. The possibility of the HRM to predict the critical mass-flow rates and the pressure distributions for one-dimensional flashing water flow has been validated. Although the model involves the use of several correlations (friction, heat transfer and the relaxation time), no adjustment of parameters against the data used for the comparison has been needed. Copyright © 1996 Elsevier Science Ltd.

Key Words: two-phase flow, flashing, thermal non-equilibrium, choked flow, metastable liquid, critical velocity

1. INTRODUCTION

The most important feature of flashing liquid flows seems to be the non-equilibrium vapour generation process as the pressure drops. It manifests itself by the liquid's failure to begin evaporation when saturation conditions are reached, leading to metastable conditions. As a consequence, flashing starts with some delay and the real quality pattern may essentially differ from the equilibrium one. This greatly influences the void fraction as well as the pressure and velocity distribution along a flow. As a result, the classical homogeneous equilibrium model (Bilicki & Kestin 1990), assuming equilibrium vapour generation, fails to reproduce not only quantitatively but also qualitatively measured distributions of flow parameters.

The so-called "slip models" are amongst the oldest two-velocity models. They assume saturation conditions for both liquid and vapour. The additional closure law for the velocity ratio is added. The slip models with Bankoff's and drift-flux closures were analysed in detail by Bilicki *et al.* (1988). All models calculated a very large increase in the void fraction as the critical state near the throat is approached. This was interpreted to mean that none of the two closures should be used to analyse critical flows, because their validity has been tested only for much lower values of void fraction.

Several elaborate computational codes (TRAC, RELAP, etc.) have been developed to satisfy the needs of nuclear-energy technology. All of them are based on the one-dimensional, two-fluid model. The success of these industrial codes is frequently due to local adjustments, which cause them to correctly reproduce limited cases. However, their predictive value for the case of two-phase critical flows does not seem to be satisfactory (Hewitt *et al.* 1987, 1990). Finally, it is necessary to mention the model recently developed by Yang (1991). It is composed of eight equations with complex relations for nucleation, evaporation rate and momentum transfer included. In spite of potential superiority, two-fluid models pose very severe closure problems. They are much more complicated and require a system of empirical closure conditions, i.e. a large number of adjustable numerical constants. Some of these constants may significantly influence the solution and they cannot always be easily determined with the required precision.

The potential usefulness of the homogeneous relaxation model (HRM) for the description of one-component, two-phase flows has been suggested by Bilicki & Kestin (1990). The choice of this model was justified not merely by its simplicity, but by the hope that, properly developed, it may prove adequate for many practical applications. Its principal advantage is its ability to account for

dispersion and dissipation of linear waves much more simply than the two-fluid model. This is confirmed with reference to the analysis of the two-fluid model of Adron & Duffey (1978). The most important dispersive phenomena were found to result from the disequilibrium between the liquid and vapour phases, combined with the fact that the liquid becomes metastable before boiling, that is from the finite rate of interphase mass transfer. The non-equilibrium interphase mass transfer is described here by means of a relaxation equation of the kind successfully exploited by Einstein (1920), Mandelshtam & Leontovich (1937), Meixner & Reik (1959) Broer (1958, 1970) and many others. To close the HRM in this form, a relation for the relaxation time corresponding to the fast evaporation (flashing) process is needed. The present paper offers a possible closure relation, being a correlation for the relaxation time calculated based on the Moby Dick (Reocreux 1974) experiments.

2. BASIC EQUATIONS

The homogeneous relaxation model (HRM) consists of the usual three conservation laws for a two-phase mixture, supplemented by the vapour mass balance equation. The basic equations are as follows:

$$\frac{\partial \rho}{\partial t} + w\frac{\partial \rho}{\partial z} + \rho\frac{\partial w}{\partial z} = -\rho w\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}z} \tag{1a}$$

$$\frac{\partial x}{\partial t} + w\frac{\partial x}{\partial z} = \frac{\Gamma}{\rho} \tag{1b}$$

$$\rho\frac{\partial w}{\partial t} + \rho w\frac{\partial w}{\partial z} + \frac{\partial P}{\partial z} = -\tau - \rho g\cos\varphi \tag{1c}$$

$$\rho\frac{\partial h}{\partial t} + \rho w\frac{\partial h}{\partial z} - \frac{\partial P}{\partial t} - w\frac{\partial P}{\partial z} = Q + w\tau \tag{1d}$$

where the total specific volume and specific enthalpy of the mixture may be defined as

$$v = xv_{\mathrm{SG}}(P) + (1-x)v_{\mathrm{ML}}(P, h_{\mathrm{ML}}) \tag{2a}$$

$$h = xh_{\mathrm{SG}}(P) + (1-x)h_{\mathrm{ML}} \tag{2b}$$

where w denotes the mixture velocity, ρ is the density of the mixture and x is the actual dryness fraction. Γ represents the vapour generation rate. Subscripts "S" and "M" denote, respectively, saturated and metastable conditions.

The preceding model neglects the slip between the phases since we assume slip to be of secondary importance to the non-equilibrium effects, $T_{\mathrm{L}} \neq T_{\mathrm{S}}(P)$. Equations [1] and [2] are very simple, and for adiabatic flow require only two additional constitutive equations for the wall friction τ and the interfacial mass transfer Γ. Let us note that if a liquid is assumed to be in saturation conditions then [1b] may be eliminated and as a result HRM reduces itself to the classical HEM.

According to any hyperbolic model, flow is choked when the smallest characteristic velocity is equal to zero. This condition gives a relation for the critical velocity. Figure 1 presents the critical velocities arising from the HEM and HRM as functions of void fraction. We can see that, for the HRM, the critical velocity tends toward the speed of sound of a subcooled liquid when the void fraction goes to zero. This is not the case for the HEM, which exhibits a large discontinuity between the critical velocities for subcooled and saturated water. This fact, apart from being physically significant, appears also to be important for numerical calculations.

3. THE RATE EQUATION

The onset of evaporation in a superheated liquid essentially changes the temperature and the total density of the two-phase mixture and, as a consequence, all the other flow parameters. Thus, the vapour generation rate Γ plays a fundamental role in [1]. This function describes a complicated process of fast non-equilibrium evaporation (flashing), including homogeneous and heterogeneous nucleation and may depend not only on the flow parameters but also on the number and structure

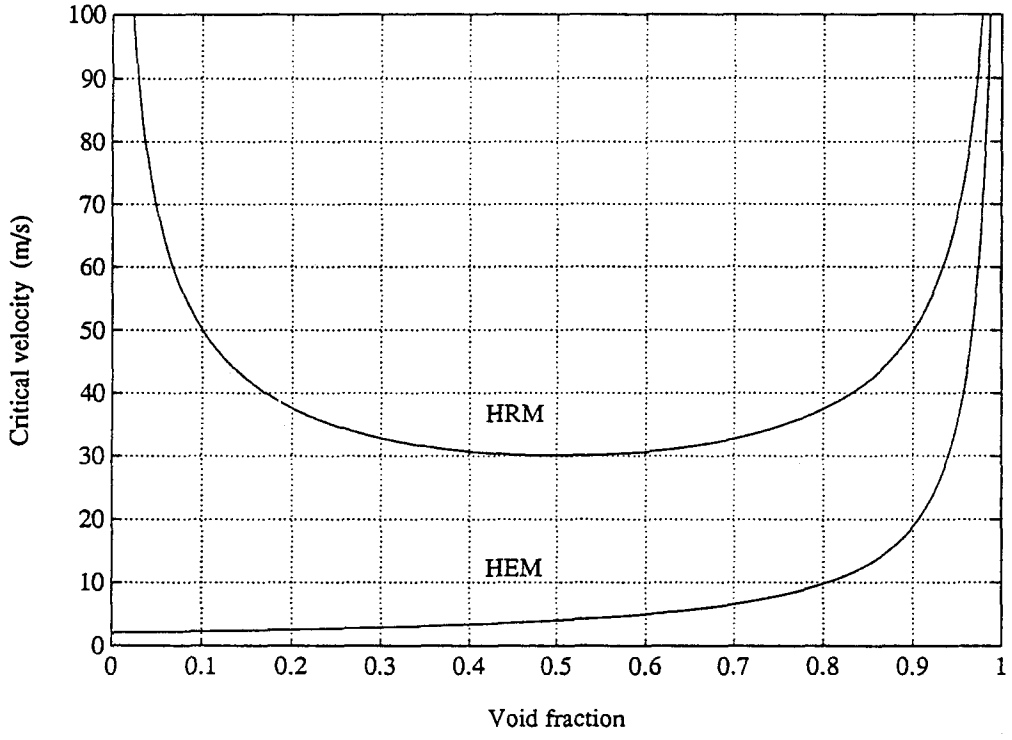

Figure 1. The critical velocity for water ($P = 2$ bar) predicted by equilibrium (HEM) and relaxation (HRM) models for the saturated conditions.

of pre-existing interfaces (liquid–gas or liquid–solid) in the metastable liquid and on the flow patterns. Being conscious of the complexity of the problem and knowing that the vapour mass production rate vanishes when the dryness fraction reaches its unconstrained equilibrium value $\bar{x}(P, h)$, we feel it relevant to adopt the relaxation equation as the simplest linear approximation, or, if the reader prefers, the first term in a Taylor series expansion of the function Γ/ρ (Bilicki & Kestin 1990). Then, [1b] takes the form

$$\frac{Dx}{Dt} = \frac{\Gamma_G}{\rho} = -\frac{x - \bar{x}}{\Theta} \qquad [3]$$

which was successfully used by Einstein (1920), Mandelshtam & Leontowich (1937) and many others. It appears that Bauer *et al.* (1976) were among the first to make use of this in the context of two-phase flow. It is not difficult to show that, locally and instantaneously, the relaxation equation [3] builds into the system an exponential tendency toward an unconstrained equilibrium from an initial composition x_0 according to

$$x = \bar{x} - (\bar{x} - x_0)\exp\left(\frac{-t}{\Theta}\right). \qquad [4]$$

Evidently, the time resolved evolution, $x(t)$, is governed by the solution of the four equations of [1]. Taking the equilibrium dryness fraction as

$$\bar{x} = \frac{h - h_{SL}(P)}{h_{SG}(P) - h_{SL}(P)} \qquad [5]$$

we can express the rate equation [3] in the form

$$\frac{Dx}{Dt} = \frac{\Gamma_G}{\rho} = (1 - x)\frac{h_{ML} - h_{SL}}{h_{SG} - h_{SL}}\frac{1}{\Theta}. \qquad [6]$$

Using this comparatively simple relation, instead of a detailed analysis of all the complicated phenomena that accompany the flashing process, may seem to model this complex situation somewhat crudely. Nevertheless, the simplicity and clear physical interpretation may promote favour of this approach.

4. THE RELAXATION TIME

In order to use [1]–[3], we need a relation for the local relaxation time Θ. Evidently, Θ could not be measured and no reliable expression existed to determine it. Bilicki *et al.* (1990) showed that the correlation given by Bauer *et al.* (1976), apart from not being dimensionally homogeneous,

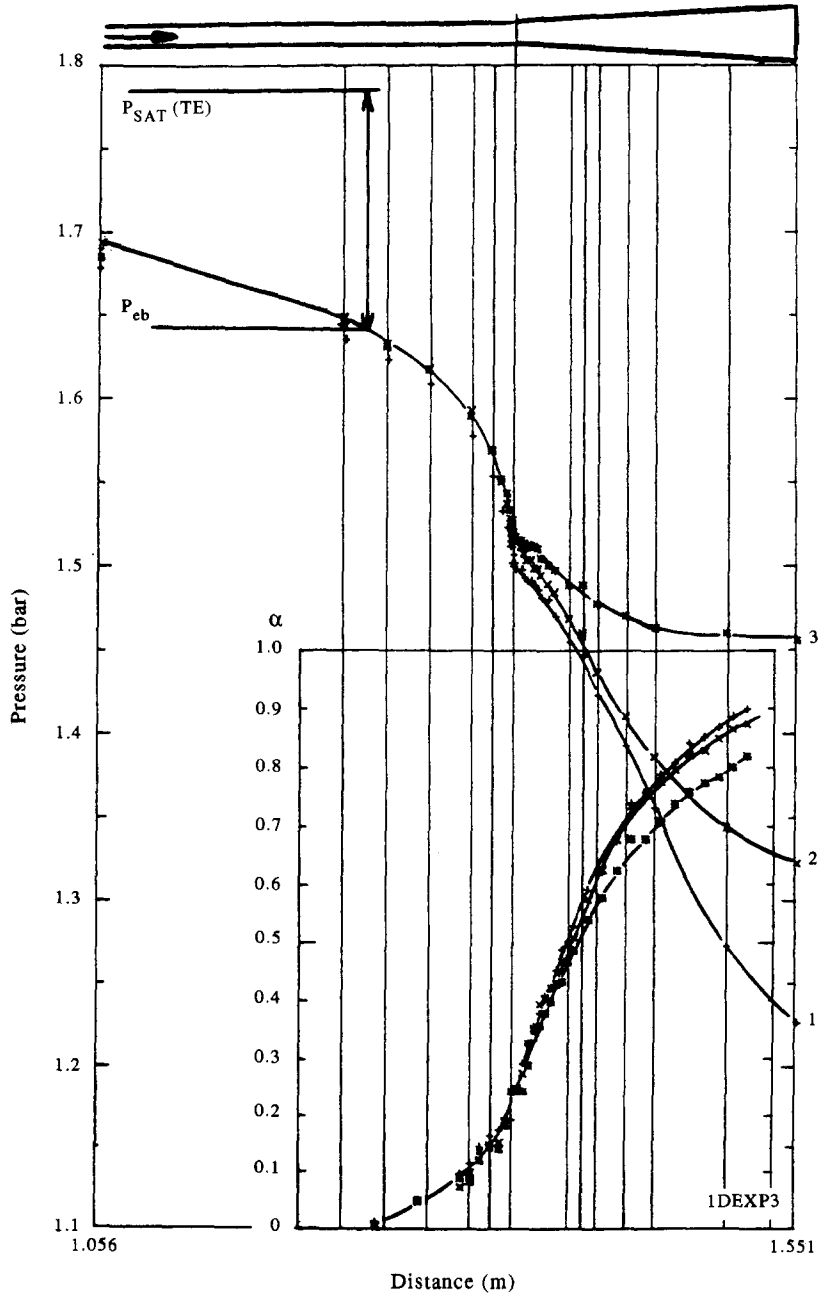

Figure 2. Typical result. (1) Run 400, $G = 6526\ \text{kg/m}^2\text{s}$, $T_{in} = 116.7°\text{C}$; (2) run 401, $G = 6465\ \text{kg/m}^2\text{s}$, $T_{in} = 116.6°\text{C}$; (3) run 402, $G = 6496\ \text{kg/m}^2\text{s}$, $T_{in} = 116.7°\text{C}$.

failed to reproduce the measured pressure distribution. For this reason, we have undertaken a trial to determine the local relaxation time based on the classical measurements of critical flow rates and of their dependence on the pressure and void fraction distribution performed by Reocreux (1974). Known as "Moby Dick" experiments, these experiments have served as a basis for the understanding of critical two-phase, single-component flows ever since their publication. They were carried out in a channel consisting of a straight portion followed by a conical expander provided with a 7° inclined-angle divergence. The channel shape was carefully designed with the intention that a one-dimensional mathematical description of the flows created in it should apply. The mass-flow rate, as well as the pressure and void fraction distributions as functions of longitudinal distance, were reported. Typical pressure and void fraction profiles are shown in figure 2, which represents three runs at the same upstream conditions but at difference back pressures (inlet diameter 20 mm). The variation of void fraction was measured with an X-ray densitometer.

Knowing the mass flux and the pressure profile, $P(z)$, we can calculate the corresponding velocity and total density distributions in the channel by solving the mass ([1a]) and momentum balance ([1c]) equations. Then the enthalpy of the liquid–vapour mixture may be calculated based on the state equation [2] and the measured values of void fraction. This way, by eliminating the dryness fraction

$$x = \frac{\epsilon v}{v_{SG}(P)} \qquad [7]$$

from [1b] in the steady-state form, we can use it to determine the local relaxation time

$$\frac{1}{\Theta(z)} = -\frac{w(z)}{x - \bar{x}}\left[\frac{\epsilon(z)}{v_{SG}(P)}\frac{dV}{dz} + \frac{v}{v_{SG}(P)}\frac{d\epsilon}{dz} - \frac{\epsilon(z)}{v_{SG}(P)}\frac{v}{v_{SG}(P)}\frac{dv_{SG}}{dP}\frac{dP}{dz}\right]. \qquad [8]$$

For the calculations, experimental data were approximated by the spline polynomials of the third order. The friction factor f was determined based on the data coming from the subcooled water flow region.

Variations of the relaxation time $\Theta(z)$ were calculated for all runs of the Moby Dick experiments for which flashing inception was situated in the test section. In all cases, the relaxation time Θ

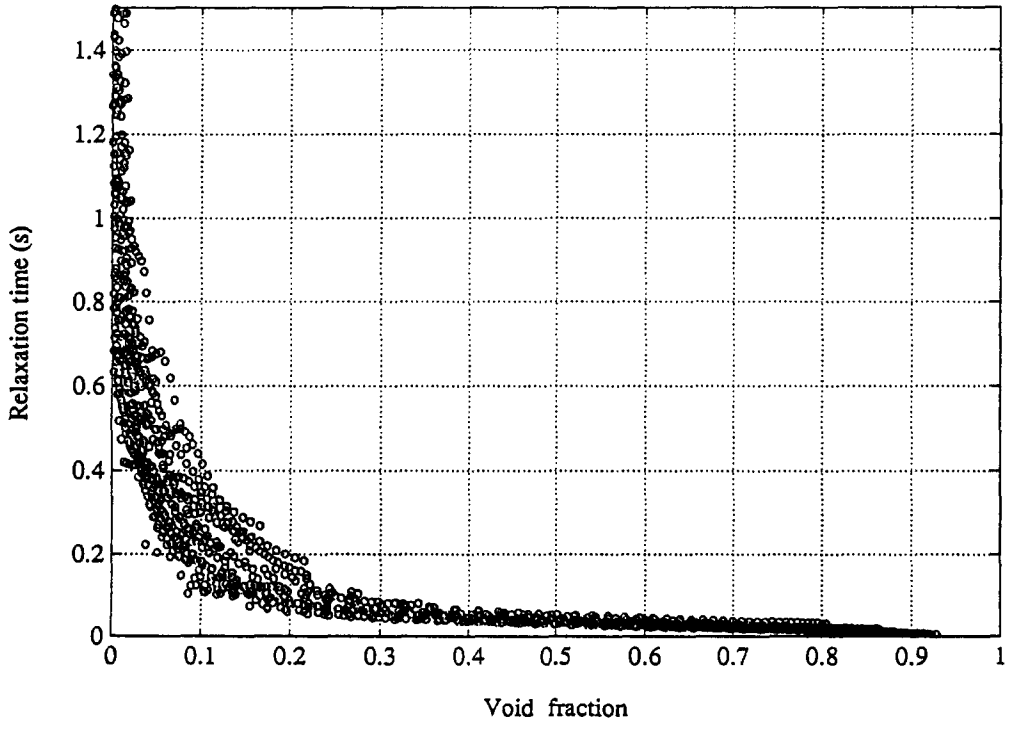

Figure 3. Calculated values of the relaxation times as a function of void fraction ϵ.

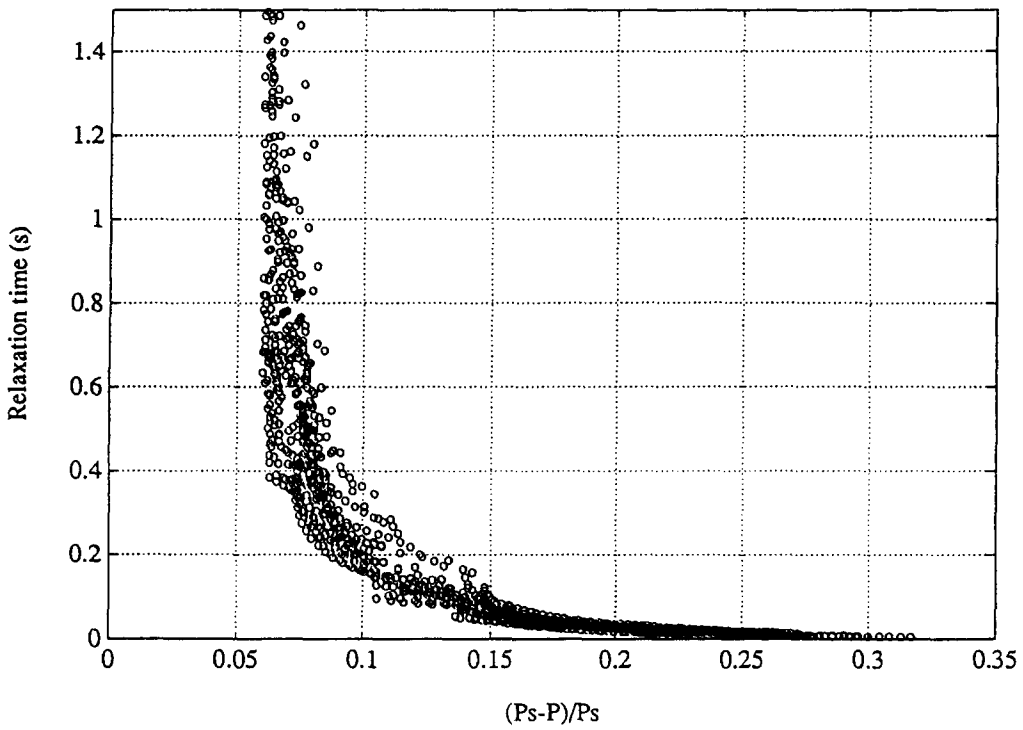

Figure 4. Calculated values of the relaxation time as a function of relative pressure drop ϕ.

appeared to be a monotonically decreasing function of void fraction ϵ and the non-dimensional pressure difference $\psi = [P_S(T_{in}) - P]/P_S(T_{in})$, taking the values of the order of 1 s at the beginning of flashing and decreasing to values of the order of 0.01 s for large void fractions. The calculated values of Θ_n as a function of ϵ_n and ψ_n are presented in figures 3 and 4. The curves $\Theta(\epsilon)$ change slope near $\Theta = 0.1$ s, which corresponds to ϵ near 0.25, where we can expect the transition in the flow regime from bubble to plug or froth flow.

The logarithmed values of Θ_n, ϵ_n and ψ_n were approximated by a plane using the least square method. Thus, the correlation for the relaxation time took the form

$$\Theta = \Theta_0 \epsilon^{-0.257} \psi^{-2.24} \quad [9]$$

where $\Theta_0 = 6.51 \times 10^{-4}$ has the dimension of time in seconds. The calculated values of Θ_n appeared to be strongly dependent on the measured void fraction distribution. They were significantly (up to 10%) influenced even by the choice of method of approximation of the experimental data of ϵ_n. The scattering of Θ_n against the correlation [9] is demonstrated in figure 5.

The correlation [9] gives comparatively good results for small pressures (up to 10 bar). For greater pressures, another form of non-dimensional pressure difference is suggested.

$$\varphi = \left[\frac{P_S(T_{in}) - P}{P_c - P(T_{in})}\right] \quad [10]$$

where P_c is the pressure of the critical point. This form of non-dimensional pressure difference was applied by Feburie *et al.* (1993). Then the correlation for the relaxation time takes the form

$$\Theta = \Theta_0 \epsilon^{-0.54} \varphi^{-1.76} \quad [11]$$

where $\Theta_0 = 3.84 \times 10^{-7}$ s.

The Moby Dick experiments revealed that the process of heterogeneous nucleation (flash point) occurs at a temperature which exceeds the saturation temperature by as much as 2–3°C. The values of Θ_n were calculated based on the measured distribution of the void fraction. In the region of superheated liquid flow (before flashing), the void fraction was too small to be measured, so the values of Θ_n could not be calculated. Nevertheless, we may attempt to use correlations [9]–[11] as

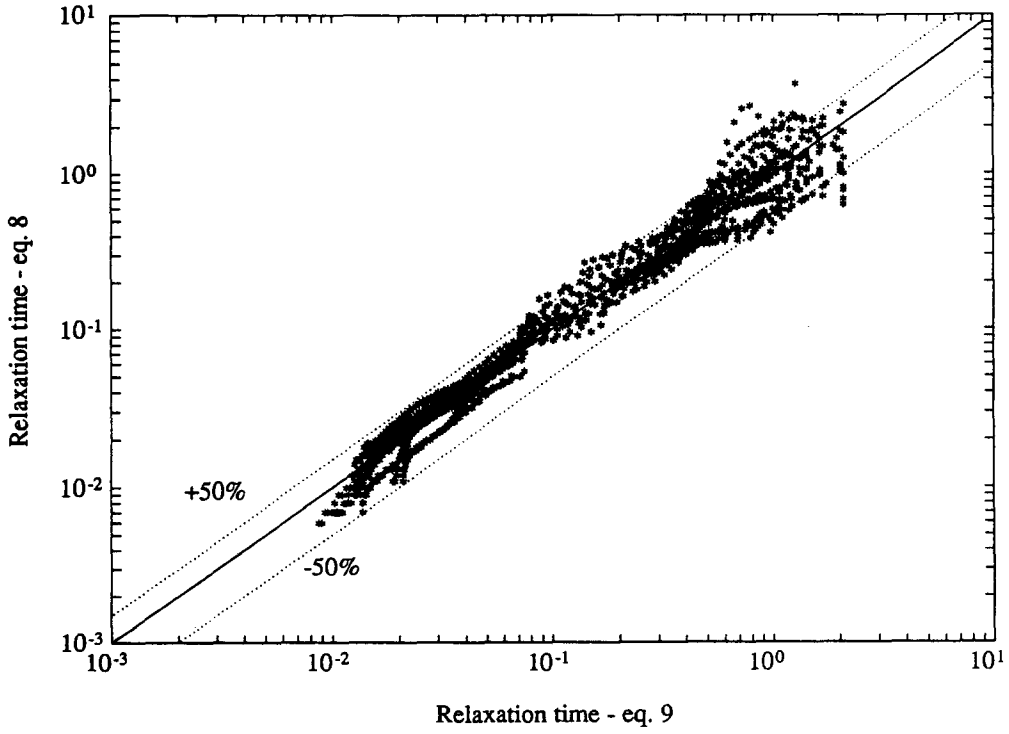

Figure 5. Comparison between the values of Θ calculated in terms of [8] and [9].

an extrapolation in this region, where vapour bubbles do not exist or they are too small to be detected.

From [3] we see that for large values of Θ the interfacial mass transfer disappears and [1] and [3] reduce to the homogeneous frozen model (HFM). On the other hand, very small values of Θ impose $x = \bar{x}$ which corresponds to HEM as the second limiting case. Numerical experiments show that, in practice, the predictions of the HRM for $\Theta = 1$ s and $\Theta = 0.001$ s are the same as for the HFM and HEM, respectively. The values of Θ_n obtained based on the Moby Dick experiments (see figures 3 and 4) indicate that the real "time-scale" takes intermediate values. Thus, the HRM with developed correlations of relaxation time places itself somewhere between the HFM and HEM as the limiting cases.

5. CALCULATION PROCEDURE AND RESULTS

A computer program has been developed to calculate the steady-state flashing liquid flow (critical or not) according to [1] with the relations [3]–[6] and [9] for irreversible mass transfer between metastable liquid and saturated vapour. For predicting the vapour–liquid friction pressure drop, the Lockhart–Martinelli correlation modified by Richardson (1958) has been used

$$\phi^2 = (1 - \epsilon)^{-1.75} \qquad [12]$$

where ϕ^2 is the ratio between the vapour–liquid friction pressure gradient and the single-phase friction pressure gradient where the liquid flows at the same flow rate as in the multiphase flow.

In the model equations [1], we have written on the r.h.s. of the energy equation the term Q related to the heat transfer between the single- or two-phase fluids and the wall. In the discussed applications, we have assumed that the flow is adiabatic, so the heat transfer term at the wall vanishes. However, there are no theoretical objections to replace it with some appropriate heat transfer law if diabatic flow has to be modelled. The thermodynamic properties of the metastable water come from extrapolation from the subcooled water region by means of the functions developed by Bilicki & Kardas (1991). The saturation vapour properties were calculated using

Garland & Hoskins' (1988) correlations. The steady-state version of the system of [1], with the above-described closure laws, has been integrated step by step using a Runge–Kutta method of fourth order. The integration is continued until either the critical flow conditions or the outlet pressure are reached. The critical flow condition (Bouré *et al.* 1976) is where the determinant of the set of model equations vanishes. The PIF (possible–impossible flow) procedure (Yan *et al.* 1991) is used to determine critical mass-flow rates. Since [1], [3] and [9] automatically reduce to the subcooled liquid model when the void fraction (dryness factor) is equal to zero, calculations may start from subcooled as well as two-phase conditions.

The model [1]–[3], with the developed correlations, has been verified against the Reocreux data (Downar-Zapolski 1993). The calculated mass-flow rates and the pressure distributions appeared to be very sensitive to the values of the relaxation time. Another comparison has been made with the experiments concerning the flashing water flow through the nozzle of the safety valve (CROSBY 1D2 JLT-JOS-15-A) made by Bolle *et al.* (1994). The inlet pressure and temperature of the initially subcooled water were kept constant. The lift of the disc was fixed high enough so that it did not influence the values of the critical mass-flow rates. Due to controlled, slow decrease of outlet pressure the critical flow conditions in the valve nozzle could be established. The PIF procedure has been used to calculate both the pressure profiles and the values of the critical mass-flow rates. Figures 6 and 7 show typical results obtained by means of the HRM. The nozzle profile is drawn on the bottom of each figure. The asterisks correspond to the measured values. It can be noted that the pressure distribution is comparatively well reproduced.

The HEM does not take the form of the subcooled water model (SWM) for the void fraction equal to zero, so before the saturation pressure was obtained the SWM had been used. For the HEM, the critical velocity does not tend toward the speed of sound of the subcooled liquid when the void fraction goes to zero, taking, for water, values of the order of 1 m/s for small void fractions (see figure 1). The velocities in the nozzle in the subcooled water flow region were always higher than 10 m/s. As a result, during calculations of the critical mass-flow rates, when we changed the model at the saturation pressure, we immediately entered a supercritical flow regime for the HEM. Thus, according to the HEM, the flashing water flows were choked exactly at the same moment when they reached the saturation pressure. In other words, for this model, subcritical flow was

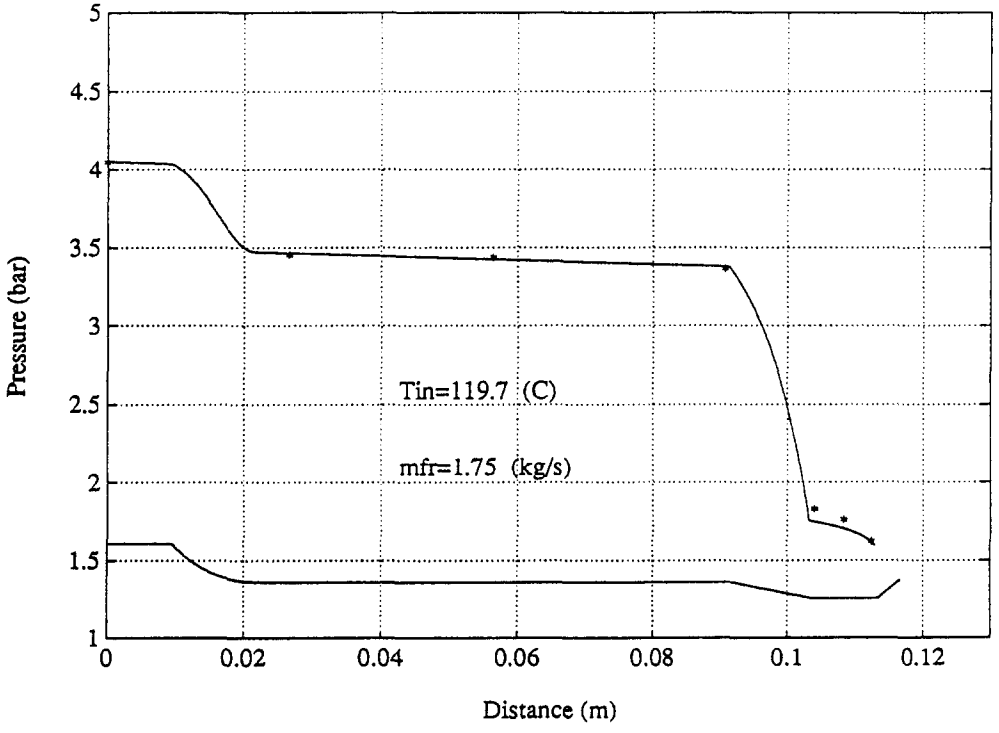

Figure 6. Pressure profiles: *, Data from Bolle *et al.* (1994); —, present model; run 11/2.

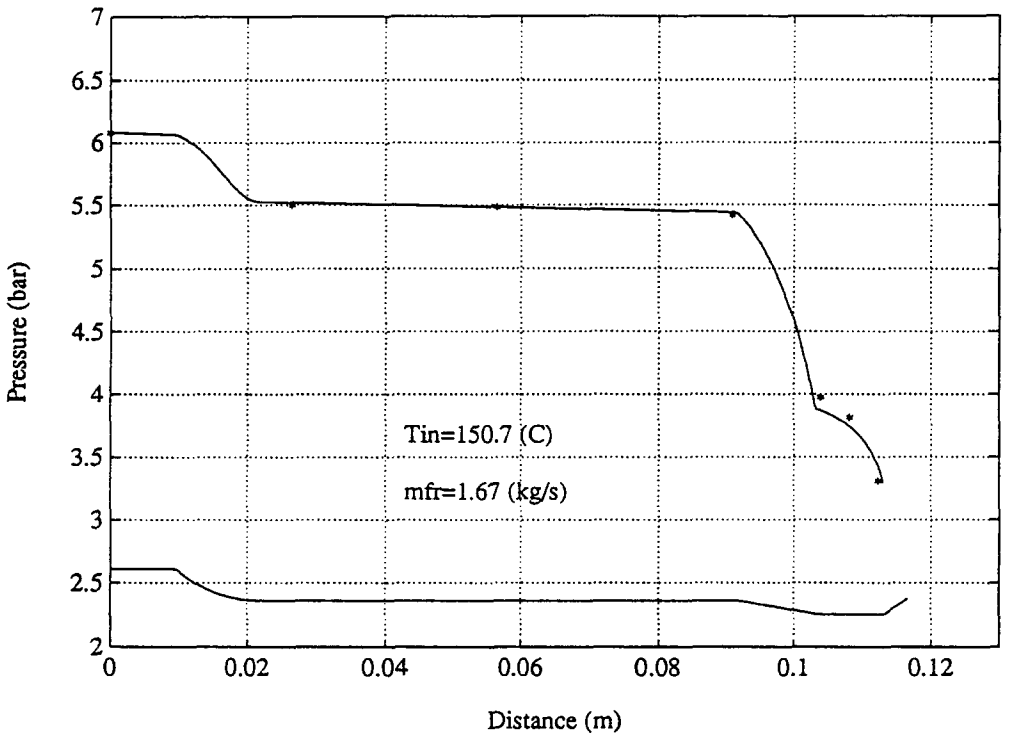

Figure 7. Pressure profiles: *, Data from Bolle *et al.* (1994); —, present model; run 15/2.

possible only when it was subcooled everywhere (pressure did not drop below the saturation pressure). This made the calculations of the pressure distribution impossible in the two-phase flow region. The pressure profile in the two-phase flow region may be calculated by means of the HEM only when the local velocity that corresponds to the saturation pressure is lower than the HEM critical velocity for the void fraction equal to zero. This condition is not always satisfied.

Table 1 presents the comparison between the critical mass-flow rates measured and predicted by the HRM and the HEM. It can be seen that the HEM generally tends to underestimate the values of the critical mass velocities. Comparatively good agreement appears only for high subcooling ($>15°C$) of inlet water, which, as the back pressure decreases, causes large nozzle velocities (≈ 20 m/s). In this case, the assumption that the flow is choked when the pressure in the nozzle is equal to that of saturation seems to be a good approximation.

Table 1. Comparison between the data from Bolle *et al.* (1994) and the present model

Run No.	P_{in} (bar)	T_{in} (°C)	$T_{sat}(P_{in}) - T_{in}$ (°C)	mfr (kg/s)	ΔHRM (%)	ΔHEM (%)
16/2	5.02	149.9	2.10	1.31	−2.6	−53.3
51/1	5.28	149.5	4.41	1.36	0.7	−36.3
54/1	4.05	138.5	5.59	1.23	−0.7	−29.0
17/2	5.60	149.7	6.47	1.61	−4.6	−32.0
72/1	5.56	149.3	7.00	1.50	1.2	−26.4
73/1	5.56	148.4	7.49	1.50	1.8	−24.4
15/2	6.08	150.7	8.66	1.67	0.7	−22.5
18/2	6.14	149.2	10.05	1.77	−0.9	−19.8
55/1	4.70	137.6	11.94	1.49	4.5	−10.2
50/1	4.68	135.6	13.78	1.50	5.0	−7.1
11/2	4.05	119.7	24.39	1.75	0.5	−2.4
12/2	5.12	119.7	33.64	2.12	0.7	0.4
7/2	5.56	120.4	35.70	2.21	1.2	1.3
14/2	5.72	119.1	37.88	2.27	1.5	1.5
8/2	6.10	120.0	39.49	2.35	1.7	1.9

6. CONCLUSIONS

A correlation for the relaxation time, being a closure relation for the homogeneous relaxation model (HRM) analysed by Bilicki & Kestin (1990), is proposed. It gives the characteristic time for the non-equilibrium evaporation process as a function of void fraction and non-dimensional pressure difference. The HRM as well as the classical HEM have been implemented in a computer program. The results are compared with some data obtained by Bolle *et al.* (1994). The non-equilibrium character of fast evaporation and its substantial influence on the two-phase flow behaviour have been noticed.

The possible–impossible flow (PIF) procedure is found to be a good way to calculate the choked two-phase flows. It enables prediction of not only the critical mass-flow rate but also the distribution of all the parameters upstream of the critical section of a choked flow. Using the PIF procedure, we are able to use any two-phase flow model, so all the assumptions that have been made can be clearly seen in the model equations. This is elegant from the physical point of view.

The HEM tends to underestimate the values of critical mass-flow rates by more than 20%, when the inlet subcooling is not large. Because of the large discontinuity of the critical velocity between the subcooled liquid model and the HEM, the calculation of the flow parameters in the two-phase region may be impossible when the fluid velocity is high.

For the HRM the critical velocity tends toward the speed of sound of pure liquid when the void fraction goes to zero. The model seems to predict, with good accuracy, both the pressure profiles along a nozzle and the critical mass-flow rates for the flashing water flow.

Finally the authors would like to emphasize that the correlation presented here may serve only as a first approximation for the relaxation time and that it was verified only for water flows. We hope that other workers in the field will reopen this problem and will direct their efforts to obtain a more physically motivated relation that could be valid for all the liquids.

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